THE FREDHOLM ALTERNATIVE FOR THE p-LAPLACIAN IN EXTERIOR DOMAINS. PART I

KY HO
NTIS, UNIVERSITY OF WEST BOHEMIA
TECHNICKÁ 8, 306 14 PLZEŇ, CZECH REPUBLIC

Abstract. In this talk we study the behavior of the energy functional associated with the following resonant problem

$$\begin{cases} -\Delta_p u = \lambda_1 K(x)|u|^{p-2}u + h & \text{in } B_1^c, \\ u = 0 & \text{on } \partial B_1, \end{cases} \tag{1}$$

where $\Delta_p u := \operatorname{div}\left(|\nabla u|^{p-2}\nabla u\right)$ is the p-Laplacian with $p > 1$, B_1^c is the complement of the closed unit ball B_1 in $\mathbb{R}^N$ ($N \geq 2$), the weight K and the function h are chosen appropriately, λ_1 is the first eigenvalue of of $-\Delta_p$ in B_1^c relative to the weight K. Similar to known results for resonant problems on a bounded domain or on the entire space $\mathbb{R}^N$ we show that the energy functional associated with (1) has "a saddle point geometry" when $1 < p < 2 \leq N$. On the other hand, we prove an improved Poincaré inequality and show that the energy functional has a "global minimizer geometry" when $2 \leq p < N$. The behavior of the energy functional will be used to obtain the existence of solutions to problem (1). The striking difference between our case and the entire space case is also discussed.

This is a joint work with Pavel Drábek and Abhishek Sarkar (Department of Mathematics and NTIS, University of West Bohemia, Czech Republic).