Blow up phenomena for Liouville systems

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We consider the following general system of two Liouville-type PDEs:

$$\begin{cases} -\Delta u_1 = 2\lambda_1 e^{u_1} - a\lambda_2 e^{u_2} & \text{in } \Omega \\ -\Delta u_2 = 2\lambda_2 e^{u_2} - b\lambda_1 e^{u_1} & \text{in } \Omega \\ u_1 = u_2 = 0 & \text{on } \partial\Omega \end{cases},$$

where $\Omega \subset \mathbb{R}^2$ is a smooth bounded domain and $a, b, \lambda_1, \lambda_2 > 0$ are positive parameters.
Using a fixed-point argument, we construct some families of solutions which blow up as $\lambda_1, \lambda_2 \to 0$. In particular, such solutions blow up with a *tower of bubble* profile, namely they look like a superposition of rescaled entire solutions of singular Liouville equations (bubbles).