Asymptotic behavior of nodal radial solutions for Moser-Trudinger problems in the ball

Let B be the unit ball in $\mathbb{R}^2$. In 1990-1992 Adimurthi and Yadava computed the *border line* nonlinearity for the existence-nonexistence of *nodal* solutions in Moser-Trudinger problems in B. More precisely they proved the following results,

Theorem (Adimurthi and Yadava)
Let us consider the problem

$$\begin{cases} -\Delta u = \lambda u e^{u^2+|u|^\beta} & \text{in } B \\ u = 0 & \text{on } \partial B. \end{cases}$$

Then we have that,

i) if $1 < \beta < 2$ *there exists a radial solution with* k *interior zeros for any integer* $k \geq 1$ *and for any* $\lambda \in (0, \lambda_1)$*,*

ii) if $0 \leq \beta \leq 1$ *there exists* $\lambda = \lambda_{AY} > 0$ *such that for any* $0 < \lambda < \lambda_{AY}$ *there exist no solution.*

We will study the asymptotic behavior of the radial solution u_ε with k nodal zeros of the problem

$$\begin{cases} -\Delta u = \lambda u e^{u^2+|u|^{1+\varepsilon}} & \text{in } B \\ u = 0 & \text{on } \partial B. \end{cases}$$

with $0 < \lambda < \lambda_{AY}$ and $\varepsilon \to 0$. This is a joint result with Daisuke Naimen (MIT, Hokkaido, Japan).