

The Fujita equation  $u_t - \Delta u = |u|^{p-1}u$  is arguably the simplest semilinear parabolic PDE. Despite its apparent simplicity it displays a staggering complexity of blow-up behaviours. Their complete description would provide an instructive glimpse into the singularity formation process in parabolic equations more generally. Such description however seems to be currently out of reach. In my talk I will discuss a selection of pressing issues in the blow-up theory for the Fujita equation and outline how some of them relate to basic questions in the theory of the stationary problem  $-\Delta u = |u|^{p-1}u$ .