Solutions to overdetermined elliptic problems in nontrivial exterior domains.

Overdetermined elliptic systems of the form

$$\begin{cases} \Delta u + f(u) &= 0 \quad \text{in} \quad \Omega \subset \mathbb{R}^n, n \geq 2 \\ u &= 0 \quad \text{on} \quad \partial\Omega \\ \frac{\partial u}{\partial \vec{n}} &= \text{constant} \quad \text{on} \quad \partial\Omega \end{cases}$$

appear in many problems in Physics and Applied Mathematics. A surprising parallelism between such problems and constant mean curvature surfaces has been observed in the last years, and this allowed to obtain some strong classification results, and some nontrivial and unexpected solutions.
In this talk, I will present the construction of new solutions in the case where the PDE is the Nonlinear Schrödinger Equation and the domain Ω is a nontrivial exterior domain, i.e. the complement of a compact region that is not the ball. These new solutions are the first examples that, up to our knowledge, have no clear counterpart in the theory of constant mean curvature surfaces. They provide also an answer, with a counterexample for all dimension $n \geq 2$, to a conjecture by Berestycki, Caffarelli and Nirenberg, which was still open in dimension 2.
This is a joint work with A. Ros and D. Ruiz.