FUJITA'S BLOWUP PROOF REVISITED

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ABSTRACT. We discuss sufficient conditions for blowup of nonnegative solutions of the Cauchy problem for the semilinear heat equation $u_t = \Delta u + u^p$ in $\mathbb{R}^d$ and radially symmetric solutions of chemotaxis systems with fractional diffusion $u_t = -(-\Delta)^{\alpha/2}u + \nabla\cdot(u\nabla v)$, $\Delta v + u = 0$, $\alpha \in (0,2]$. The proof of blowup follows the original H. Fujita argument analyzing the moment $\int G(x,t)u(x,t)dx$ where G solves the backward heat equation $G_t + \Delta G = 0$ in $\mathbb{R}^d \times (0,T)$ with the final condition $G(.,T) = \delta_0$. A sufficient condition for the blowup is then interpreted in terms of a critical Morrey space norm, either $M^{d(p-1)/2}(\mathbb{R}^d)$ or $M^{d/\alpha}(\mathbb{R}^d)$, of the initial condition u_0. On the other hand, if this norm of u_0 is small, the solution is global in time.

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