Periodic solutions of the point vortex Hamiltonian system

T. Bartsch
University of Giessen

Abstract: The dynamics of N point vortices $z_1, \dots, z_N$ in a bounded domain $\Omega \subset \mathbb{R}^2$ is described by the the Hamiltonian system

$$\Gamma_k \dot{z}_k = \nabla_{z_k}^{\perp} H(z_1, \dots, z_N)$$

with Hamiltonian

$$H(z_1, \dots, z_N) = -\frac{1}{2\pi} \sum_{\substack{j,k=1 \\ j \neq k}}^{N} \Gamma_j \Gamma_k \log |z_j - z_k| - \sum_{j,k=1}^{N} \Gamma_j \Gamma_k g(z_j, z_k).$$

Here $g : \Omega \times \Omega \to \mathbb{R}$ is the regular part of the Green's function of the Dirichlet Laplacian in Ω, and $\Gamma_k \neq 0$, $k = 1, \dots, N$, are real parameters. The system arises as a limit of the Euler equation in vorticity form when the vorticity concentrates at N points in the domain.

We prove the existence of periodic solutions of the system that are localized near critical points of the Robin function $h(x) = g(x, x)$. After a blow-up these solutions look like a periodic solution of the point vortex Hamiltonian system in the plane. Using a degree theory for S^1-equivariant potential operators we also show that our solutions lie on a global continuum of periodic solutions.

This is joint work with Qianhui Dai and Björn Gebhard.