2D QUASI-GEOSTROPHIC EQUATION; SUB-CRITICAL AND CRITICAL CASES

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We are using a version of the 'vanishing viscosity technique' to study the Dirichlet problem for *critical 2D Quasi-Geostrophic equation*:

$$\begin{aligned} &\theta_t + u \cdot \nabla\theta + \kappa(-\Delta)^\alpha \theta = f, \quad x \in \Omega \subset \mathbb{R}^2,\ t > 0, \\ &\theta(0,x) = \theta_0(x), \end{aligned} \tag{1}$$

where θ represents the potential temperature, $\kappa > 0$ is a diffusivity coefficient, $\alpha \in [\frac{1}{2}, 1]$ a fractional exponent, and $u = (u_1, u_2)$ is the *velocity field* determined by θ through the relation:

$$u = \mathcal{R}^\perp \theta \quad \text{with } \mathcal{R} = \nabla(-\Delta)^{-\frac{1}{2}}. \tag{2}$$

Solving first the *sub-critical approximating equations* (1) with $\alpha \in (\frac{1}{2}, 1]$, we let then $\alpha \to \frac{1}{2}$ to obtain a solution corresponding to the critical value of exponent $\alpha = \frac{1}{2}$. We discuss next in some details properties of solutions of the critical problem; in particular their uniqueness, regularity and long time behavior. The lecture is based on the recent publication [1], joint with Chunyou Sun, and uses the technique presented in [2].

References

[1] T. Dlotko, C. Sun, *2D Quasi-Geostrophic equation; sub-critical and critical cases*, Nonlinear Analysis 150 (2017), 38-60.

[2] T. Dlotko, *Navier-Stokes equation and its fractional approximations*, Appl. Math. Opt., DOI 10.1007/s00245-016-9368-y (former title *New look at the Navier-Stokes equation*).

[3] T. Dlotko, M.B. Kania, C. Sun, *Quasi-geostrophic equation in* R^2, J. Differential Equations 259 (2015), 531-561.

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