

ON MULTIPLICITY OF EIGENVALUES AND SYMMETRY OF EIGENFUNCTIONS OF THE p -LAPLACIAN

VLADIMIR BOBKOV

Let B be a ball in \mathbb{R}^N , $N \geq 2$. Consider the eigenvalue problem

$$(0.1) \quad \begin{cases} -\Delta_p u = \lambda |u|^{p-2} u & \text{in } B, \\ u = 0 & \text{on } \partial B, \end{cases}$$

where $\Delta_p u = \operatorname{div}(|\nabla u|^{p-2} \nabla u)$, $p > 1$. It is well-known that a sequence of variational eigenvalues of (0.1) can be obtained by means of the following minimax variational principle:

$$\lambda_k(p; B) := \inf_{\mathcal{A} \in \Gamma_k(p)} \max_{u \in \mathcal{A}} \frac{\int_B |\nabla u|^p dx}{\int_B |u|^p dx}, \quad k \in \mathbb{N},$$

where $\Gamma_k(p)$ is a family of symmetric and compact subsets of $W_0^{1,p}(B)$ with Krasnosel'skii genus greater than or equal to k .

In the linear case $p = 2$, it is well-known that $\lambda_2(2; B) = \dots = \lambda_{N+1}(2; B) = \lambda_\ominus(2; B)$, where $\lambda_\ominus(2; B)$ is the eigenvalue of the Laplacian which has an associated eigenfunction whose nodal set is an equatorial section of B . We are interested in the generalization of this fact to the nonlinear case. One of our main results in this direction is formulated as follows.

Theorem 1. *The following chain of inequalities is satisfied:*

$$\lambda_2(p; B) \leq \dots \leq \lambda_{N+1}(p; B) \leq \lambda_\ominus(p; B).$$

If $\lambda_2(p; B) = \lambda_\ominus(p; B)$, as it holds true for $p = 2$, this result implies that the variational (algebraic) multiplicity of the second eigenvalue is at least N . Moreover, in the case $N = 2$, we can deduce from [1] that any *third* eigenfunction of the p -Laplacian on a disc is nonradial.

This is a joint work with BENJAMIN AUDOUX and ENEA PARINI (Aix Marseille Univ, CNRS, Marseille, France), see [2].

REFERENCES

- [1] J. Benedikt, P. Drábek, and P. Girg. The second eigenfunction of the p -laplacian on the disk is not radial. *Nonlinear Analysis: Theory, Methods & Applications*, 75(12):4422–4435, 2012. doi:10.1016/j.na.2011.06.012.
- [2] B. Audoux, V. Bobkov, and E. Parini. On multiplicity of eigenvalues and symmetry of eigenfunctions of the p -laplacian. *arXiv*, 1704.03194, 2017. URL: <https://arxiv.org/abs/1704.03194>.

(V. Bobkov) UNIVERSITY OF WEST BOHEMIA, FACULTY OF APPLIED SCIENCES, DEPARTMENT OF MATHEMATICS AND NTIS, UNIVERZITNÍ 8, 306 14 PLZEŇ, CZECH REPUBLIC