THE INTEGRAL REPRESENTATION FOR THE EFFECTIVE ENERGY FUNCTIONAL FOR THIN FILMS WITH BENDING MOMENT IN THE ORLICZ-SOBOLEV SPACE SETTING

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Let ω be a bounded open subset of $\mathbb{R}^2$ with Lipschitz boundary and $\Omega := \omega \times (-\frac{1}{2}, \frac{1}{2}) \subset \mathbb{R}^3$. We consider the functional $\bar{J}_\varepsilon : W^{1,M}(\Omega;\mathbb{R}^3) \times L^M(\Omega;\mathbb{R}^3) \to \mathbb{R} \cup \{+\infty\}$ defined by

$$\bar{J}_\varepsilon(u,\bar{b}) := \begin{cases} \int_\Omega W(D_1 u | D_2 u | \frac{1}{\varepsilon} D_3 u) dx & \text{if } \frac{1}{\varepsilon} D_3 u = \bar{b} \\ +\infty & \text{otherwise}, \end{cases}$$

where $W : \mathbb{R}^{3\times 3} \to \mathbb{R}$ is a continuous function satisfying the growth and coercivity conditions

$$\frac{1}{C}(M(\|F\|) - 1) \le W(F) \le C(1 + M(\|F\|)) \quad (\forall F \in \mathbb{R}^{3\times 3})$$

for some $C > 0$. Here $M\colon \mathbb{R} \to [0,\infty)$ is some non-power Orlicz-Young convex function.

We present the integral estimation for the Γ-lower and Γ-upper limits for functional $\bar{J}_\varepsilon$, by studying the equivalent re-scaled integral functional

$$\bar{F}_\varepsilon(H) := \begin{cases} \frac{1}{\varepsilon} \int_{\Omega_\varepsilon} W(H(x)) dx & \text{if curl } H = 0 \text{ in } \Omega_\varepsilon \text{ (distributionally)} \\ +\infty & \text{otherwise}, \end{cases}$$

with $H = Du \in L^M(\Omega_\varepsilon;\mathbb{R}^{3\times 3})$, $\Omega_\varepsilon := \omega \times (-\frac{\varepsilon}{2}, \frac{\varepsilon}{2})$, as the thickness ε goes to 0.

The results are extended also for the case $\mathcal{A} = \operatorname{div}$ (which appears in the context of functional on solenoidal vector fields) or $\mathcal{A} = \begin{pmatrix} \operatorname{div} & \operatorname{div} \\ 0 & \operatorname{curl} \end{pmatrix}$ (which appears in the context of micro-magnetic functional) instead of $\mathcal{A} = \operatorname{curl}$. All results were obtained in collaboration with prof. Hong Thai Nguyen from University of Szczecin.