BIFURCATION FROM INFINITY FOR AN ELLIPTIC PROBLEM IN $\mathbb{R}^N$

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The talk is based on [2] and [1]. We consider the asymptotically linear Schrödinger equation

$$-\Delta u + V(x)u = \lambda u + f(x,u), \quad x \in \mathbb{R}^n$$

as well as an abstract problem of the form

$$Lu = \lambda u + N(u),$$

where L is a linear (unbounded) operator in a Hilbert space. We show that if λ_0 is an isolated eigenvalue for the linearization at infinity, then under some additional conditions there exists a sequence (u_n, λ_n) of solutions such that $\|u_n\| \to \infty$ and $\lambda_n \to \lambda_0$. If the potential $V \in L^\infty(R^N)$, then we use degree theory if the multiplicity of λ_0 is odd and Morse theory (or more specifically, Gromoll-Meyer theory) if it is not. In the case of a possible potential-well, i.e., when $V = V_0 + V_\infty$, where $V_\infty \in L^\infty$ and is strictly positive and $V_0 \in L^p(R^N)$ with $p > N$, then our approach is based on the version of the Conley index due to Rybakowski and the existence relies on a variant of the Landesman-Lazer conditions.

References

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[3] C. A. Stuart, *Asymptotic bifurcation and second order elliptic equations on* $\mathbb{R}^n$, Ann. IHP - Analyse Non Linéaire (2014), http://dx.doi.org/10.1016/j.anihpc.2014.09.003.