

# Curve-rational functions

**Abstract.** Let  $W$  be a subset of the set of real points of a real algebraic variety  $X$ . We investigate which functions  $f : W \rightarrow \mathbb{R}$  are the restrictions of rational functions on  $X$ . We introduce two new notions: *curve-rational functions* (i.e., continuous rational on algebraic curves) and *arc-rational functions* (i.e., continuous rational on arcs of algebraic curves). We prove that under mild assumptions the following classes of functions coincide: continuous hereditarily rational (introduced recently by the first named author), curve-rational and arc-rational. In particular, if  $W$  is semialgebraic and  $f$  is arc-rational, then  $f$  is continuous and semialgebraic. We also show that an arc-rational function defined on an open set is arc-analytic (i.e., analytic on analytic arcs). Furthermore, we study rational functions on products of varieties. As an application we obtain a characterization of regular functions. Finally, we get analogous results in the framework of complex algebraic varieties. This is joint work of János Kollár, Wojciech Kucharz and Krzysztof Kurdyka.