Symplectic factorization, Darboux theorem and ellipticity

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The talk is based on the joint work with Wifrid Gangbo and Olivier Kneuss

Our first result concerns the classical Darboux theorem. We prove that if ω_m is the standard symplectic form and f is a symplectic form, then we can find a diffeomorphism φ, with optimal regularity, satisfying

$$\varphi^*(\omega_m) = f \quad \text{and} \quad \sum_{i=1}^{m} \left(\frac{\partial \varphi^{2i}}{\partial x_{2i-1}} - \frac{\partial \varphi^{2i-1}}{\partial x_{2i}} \right) = 0$$

provided that f is a small perturbation of ω_m. Moreover we show that the above system is elliptic and that we have uniqueness, when coupled with a Dirichlet datum.

We then apply the above result to the so-called symplectic factorization. We show that any map φ, satisfying appropriate assumptions, can be written as

$$\varphi = \psi \circ \chi$$

where

$$\psi^*(\omega_m) = \omega_m \quad \text{and} \quad \sum_{i=1}^{m} \left(\frac{\partial \chi^{2i}}{\partial x_{2i-1}} - \frac{\partial \chi^{2i-1}}{\partial x_{2i}} \right) = 0.$$

The analogy with mass transportation and the Monge-Ampère equation, as well as with the polar decomposition, will be emphasized.