The problem of uniqueness in the calculus of variations

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It is known that minimizers of variational integrals

$$I(u) = \int_{\Omega} F(\nabla u(x))\,\mathrm{d}x, \quad u \in \mathrm{W}^{1,2}_{g}(\Omega, \mathbb{R}^N)$$

under natural conditions on the integrand F need not be fully regular nor unique.

In this talk I discuss the questions of full regularity and uniqueness of minimizers under suitable smallness conditions on the Dirichlet boundary datum g. It is in this connection interesting to compare the results that can be obtained by use of the Implicit Function Theorem with those that can be obtained using arguments from regularity theory more directly. The last approach yields slightly stronger results, and also raises some interesting questions that involve a Gårding inequality and a Rayleigh type quotient. The Gårding inequality ensures a certain equi-coercivity of the Rayleigh quotient, and this in turn allows one to prove uniqueness and stability results for minimizers under rather weak smallness conditions on g. The validity of the Gårding inequality means that the variational integral is convex on a subspace of finite codimension in the Dirichlet class.