The Divergence Theorem in Rough Domains and Applications

Marius Mitrea

University of Missouri, USA
mitream@missouri.edu

Arguably, one of the most basic results in analysis is Gauss' Divergence Theorem. Its original formulation involves mildly regular domains and sufficiently smooth vector fields (typically both of class C^1), though applications to rougher settings have prompted various generalizations. One famous extension, due to De Giorgi and Federer, lowers the regularity assumptions on the underlying domain to a mere local finite perimeter condition. While this is in the nature of best-possible, the De Giorgi and Federer theorem still asks that the intervening vector field has Lipschitz components. The latter assumption is, however, unreasonably strong, both from the point of view of the very formulation of the Divergence Formula, and its applications to PDE's which often involve much less regular functions. In my talk I will discuss a refinement which addresses this issue, through the use of tools and techniques from Harmonic Analysis (Whitney decompositions, weighted isoperimetric inequalities, nontangential maximal operators, etc). In particular, this sharpened format of the Divergence Theorem yields very general Cauchy reproducing formulas for holomorphic functions in rough domains.