Finite distortion Sobolev mappings between manifolds are continuous

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We prove that if M and N are Riemannian, oriented n-dimensional manifolds without boundary and additionally N is compact, then Sobolev mappings $W^{1,n}(M,N)$ of finite distortion are continuous. In particular, $W^{1,n}(M,N)$ mappings with almost everywhere positive Jacobian are continuous. This result has been known since 1976 in the case of mappings $W^{1,n}(\Omega,\mathbb{R}^n)$, where $\Omega \subset \mathbb{R}^n$ is an open set. The crucial step in the proof was the use of the existence of a Lipschitz retraction of $\mathbb{R}^n$ onto a ball. Since there are no such retractions in the case of compact manifolds, the case of $W^{1,n}(M,N)$ mappings is much more difficult.

In fact, in the Euclidean case one can basically use the same argument as in the case of $W^{1,n}$ mappings to prove that if $f : \Omega \to \mathbb{R}^n$, $\Omega \subset \mathbb{R}^n$ satisfies $Df \in L^n\mathrm{Log}^{-1}$ and has finite distortion, then f is continuous. However, in the case of mappings between manifolds that have finite distortion and satisfy $Df \in L^n\mathrm{Log}^{-1}$, continuity of f depends on the topology of the target manifold. This is caused basically by the lack of the existence of a retraction of the manifold onto a ball. The result shows an interesting phenomena: both the continuity of a mapping and the finite distortion condition are local in their nature. However, whether a mapping between manifolds that has finite distortion and satisfy $Df \in L^n\mathrm{Log}^{-1}$ is continuous depends on the global properties of the target manifold.

The talk is based on two papers, one with Paweł Goldstein and Reza Pakzad and one with Paweł Goldstein.