The Jacobian problem of Coifman, Lions, Meyer and Semmes

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R. Coifman, P.-L. Lions, Y. Meyer and S. Semmes showed in their celebrated paper from 1993 that numerous compensated compactness quantities such as Jacobians of mappings in $W^{1,n}(\mathbb{R}^n, \mathbb{R}^n)$ belong the real-variable Hardy space $\mathcal{H}^1(\mathbb{R}^n)$. They proceeded to ask what is the exact range of these nonlinear quantities and in particular whether the Jacobian operator J maps $W^{1,2}(\mathbb{R}^2, \mathbb{R}^2)$ onto $\mathcal{H}^1(\mathbb{R}^2)$.

In the talk I give the proof of my recent result that $J\colon W^{1,n}(\mathbb{R}^n, \mathbb{R}^n) \to \mathcal{H}^1(\mathbb{R}^n)$ is not surjective for any $n \geq 2$. The surjectivity question is still open when the domain of definition of J is the inhomogeneous Sobolev space $\dot{W}^{1,n}(\mathbb{R}^n, \mathbb{R}^n)$. I also shortly discuss T. Iwaniec's conjecture from 1997 which states that for every $n \geq 2$ and $p \in [1, \infty[$ the operator $J\colon \dot{W}^{1,np}(\mathbb{R}^n, \mathbb{R}^n) \to \mathcal{H}^p(\mathbb{R}^n)$ has a continuous right inverse.