The story of lip

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One way of defining an infinitesimal Lipschitz constant for a function $f\colon \mathbb{R} \to \mathbb{R}$ is as follows:

$$\operatorname{lip} f(x) = \liminf_{r \to 0} \sup_{y \in B(x,r)} \frac{|f(y) - f(x)|}{r}.$$

In the same spirit as people obtained characterizations of the set where the derivative is infinite, we are interested in the set where lip is infinite. We know that given a continuous function $f\colon \mathbb{R} \to \mathbb{R}$, the set where $\operatorname{lip} f = \infty$ is an $F_{\sigma\delta}$-set; remember that $F_{\sigma\delta}$-sets are the sets that can be written as countable intersection of a countable union of closed sets. We believe that the converse is true as well: given an $F_{\sigma\delta}$-set, we can find a continuous function $f\colon \mathbb{R} \to \mathbb{R}$ such that the set of points x where $\operatorname{lip} f(x) = \infty$ is exactly F. Although we embarked with David Preiss and Martin Rmoutil on the journey to prove this statement, we did not yet reach the state that we can claim we have a proof. Hence, this poster focuses on the intermediate step of constructing a function for a given F_σ-set. This part is joint work of Zoltán Buczolich, Bruce Hanson, Martin Rmoutil and me. We hope that we soon will be able to submit it. To familiarize people with lip, a great part of the poster tells the story how I got acquainted with lip and her big brother Lip.