Compressed sensing in the quaternion algebra

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Abstract

Thirteen years ago, E. Candès et al. showed that – in the real or complex setting – if a measurement matrix satisfies so-called *restricted isometry property* (RIP) with a sufficiently small constant, then every sparse signal can be uniquely reconstructed from a limited number of its linear measurements as a solution of a convex program of ℓ_1-norm minimization. Sparsity of the signal is a natural assumption – most of well known signals have a sparse representation in an appropriate basis (e.g. wavelet representation of an image). Moreover, if the original signal is not sparse, the same minimization procedure provides a good sparse approximation of the signal and the procedure is stable in the sense that the error is bounded above by the ℓ_1-norm of the difference between the original signal and its best sparse approximation.

For a certain time the attention of the researchers in the theory of *compressed sensing* has mostly been focused on real and complex signals. Over the last decade, there have been published results of numerical experiments suggesting that the compressed sensing methods can be successfully applied also in the quaternion algebra, however, until recently there were no theoretical results that could explain the success of these experiments. The generalization of compressed sensing to the quaternion algebra would be significant due to their wide applications, e.g. in colour images processing.

In the talk we will present results of our research on developing theoretical background of the compressed sensing theory in the quaternion algebra.

References

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