Structural Topology Optimization of Variational Inequalities

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Abstract

The goal of the structural topology optimization is to find the optimal distribution of a few elastic materials in a given design domain to minimize a functional describing either the mechanical or the thermal properties of the structure or its cost. The loaded structure is assumed to satisfy the volume or mass constraints imposed on it. In recent years two or multiple phases topology optimization problems have become subject of the growing interest since it leads to improved design solutions. The paper deals with the topology optimization for systems governed by variational inequalities. This class of systems includes among others the unilateral contact phenomenon with Tresca friction between elastic solid bodies. To ensure the existence of optimal solutions the original cost functional is regularized using the multiphase volume constrained Ginzburg-Landau energy functional rather than the perimeter functional. In this model the width of the interfaces between domain material phases is dependent on a small parameter. The first order necessary optimality condition is formulated. As the interface width parameter tends to zero the Γ-convergence in L^1 of the sequence of phase field regularized functionals to the sharp interface functional is shown. Moreover the convergence of the first order necessary optimality conditions for the phase field regularized optimization problem to the first order optimality condition for the perimeter regularized optimization problem obtained in the framework of shape calculus is proved. The optimal topology is obtained numerically as the steady state of the phase transition governed by the generalized Allen-Cahn or Cahn-Hilliard equations using the operator splitting approach combined with the projection gradient method. Numerical examples are provided and discussed.

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