Controllability and Optimal Control Problems with Infinite Horizon

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Abstract

In this talk we address the stability of a general nonlinear system. Nonlinear dynamical systems are found in almost all fields of science and engineering and therefore the subject of stability analysis maintains a strong research interest. In the literature several approaches to achieve stabilization as well as many other important results about different stability concepts can be found. For example, stability results based on a so called inverse optimal control problem are introduced by Kalman. Other approaches are based on receding horizon techniques and often make use of a control Lyapunov function as terminal costs to ensure closed loop stability, cf. Ito and Kunisch. The philosophy presented here relies on studying a linear-quadratic regulator (LQR) problem with infinite horizon embedded in a natural space setting, that is characterized by a weight function $\nu(t) = e^{-\beta t}$ with $\beta \neq 0$. In particular, in this setting our problem becomes a LQR in Hilbert spaces where questions of existence and regularity of optimal solutions can be dealt with classical tools of convex analysis. Starting with a given nonlinear system, we first use standard techniques to linearize around an equilibrium point. Then the dynamic equation represented by matrices $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{m\times m}$ is given as

$$\dot{x}(t) = Ax(t) + Bu(t), \;\; x(0) = x_0 \tag{1}$$

where $x(t)$ and $u(t)$ denote the $n-$dimensional system state space vector and the $m-$dimensional control vector, respectively. To find an asymptotically stable state with $\lim_{t\to\infty} x(t) = 0$, we have to find the integrated weights $W \in \mathbb{R}^{n\times n}$ and $R \in \mathbb{R}^{m\times m}$ in the following optimal control formulation: Minimize

$$J(x,u) = \frac{1}{2}\int_0^\infty \left\{x^T(t)Wx(t) + u^T(t)Ru(t)\right\}\nu(t)dt$$

subject to all pairs $(x,u) \in W_2^{1,n}(\mathbb{R}_+,\nu) \times L_2^m(\mathbb{R}_+,\nu)$ satisfying the state equations and the initial conditions in (1) a.e. on $\mathbb{R}_+$. Using a Pontryagins's type Maximum Principle as a necessary criteria for optimality as well as transversality conditions for the adjoints, the given system can be stabilized by a suitable control action, which can be realized as the solution of the canonical equations. An example illustrates the applicability of the proposed technique.

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