

Linearizations of Polynomial Matroid Optimization Problems

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Abstract

In this talk we consider polynomial matroid optimization problems where the non-linear monomials satisfy certain monotonicity properties. Indeed, we study problems where the set of non-linear monomials consists of all non-linear monomials that can be built from a given subset of the variables. Linearizing all non-linear monomials we study the respective polytope. Extending results of Edmonds [Edm70] we present a complete description of this polytope. Apart from linearization constraints one needs appropriately strengthened rank inequalities. The separation problem for these inequalities reduces to a submodular function minimization problem. These polyhedral results give rise to a new hierarchy for the solution of general polynomial matroid optimization problems. We compare our new hierarchy to the hierarchy of Sherali and Adams [SA90, SA98]. Furthermore we show some examples where monotonicity of the monomials is not needed in order to derive a complete description of the associated linearized polytope.

References

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