Lipschitz stability of minimization problems

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Let $\mathcal{H}$ be a real Hilbert space, $\mathbb{K}$ — the set of closed convex bounded subsets from $\mathcal{H}$. Daniel [1] proved, that the metric projection of a point from $\mathcal{H}$ on a closed convex subset in the Hilbert space is Holder continuous with the power $\frac{1}{2}$ with respect to the set in the standard Hausdorff metric. Moreover, the solution of the problem

$$\min_{x\in A} f(x)$$

for a closed convex subset $A \subset \mathcal{H}$ and a strongly convex function f is also Holder continuous with the power $\frac{1}{2}$ with respect to the set in the Hausdorff metric.

Thus, the Lipschitz dependence of solution for wide class of minimization problems from the set in the Hausdorff metric is not typical. The same situation takes place for some maximization problems ($\max_{a\in A} \|x - a\|$, see [2]).

We also want to recall the result of J. Lindenstrauss [3, Cor. 5 page 271] that there is no such Lipschitz (and even uniformly continuous) function $f : \mathbb{K} \to \mathcal{H}$ that f is Lipschitz in the Hausdorff metric and $f(A) \in A$ for all $A \in \mathbb{K}$.

In the present talk we consider another metric on the space of convex closed bounded subsets from $\mathcal{H}$ instead of the Hausdorff metric, where the Lipschitz property of mentioned problems is typical.

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[1] J. W. Daniel, The continuity of metric projection as function of data, J. Approxim. Theory 12:3 (1974), 234-240.

[2] Maxim V. Balashov, Antidistance and Antiprojection in the Hilbert Space, Journal of Convex Analysis 22 (2015), No. 2, 521–536.

[3] J. Lindenstrauss, On nonlinear projection in Banach spaces, Michigan Math. J., 11:3 (1964), 263–287.