

A discrete variant of Farkas' Lemma and related results

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Abstract

We have established a discrete variant of Farkas' Lemma in the setting of a module over a linearly ordered commutative ring, which may contain zero divisors, and need not be associative nor unital [1]. However, to establish the result, we have had to assume an auxiliary hypothesis about the ring. In an addendum [2], we have proved the same result, but we have assumed another auxiliary hypothesis to prove the result. We have shown in [2] that both hypotheses about the ring (assumed in [1] and [2], respectively) are independent of each other. Hence, it has been an open question whether the discrete variant of Farkas' Lemma holds if neither of the two hypotheses is satisfied.

In this paper, we report that the discrete variant of Farkas' Lemma holds in general, i.e. no auxiliary hypothesis about the ring is necessary.

We also report the corresponding discrete variants of some related results: Tucker's Key Theorem, Motzkin's Theorem of the alternative and Tucker's Theorem of the alternative.

References

- [1] D. Bartl, D. Dubey: A discrete variant of Farkas' Lemma, *Operations Research Letters* 45 (2017) 160–163
- [2] D. Bartl: A discrete variant of Farkas' Lemma: an addendum. In: R. Němec, L. Chytilová (eds.), *Proceedings of the 12th International Conference on Strategic Management and its Support by Information Systems 2017: May 25.–26., 2017: Ostrava, Czech Republic*, VŠB – Technical University of Ostrava, Faculty of Economics, pp. 176–183. ISBN 978-80-248-4046-8. ISSN 2570-5776

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