Stability in set optimization: a set-covergence approach

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Abstract

We consider a sequence of perturbed problems with a fixed set-valued objective map converging to a given set optimization problem. We limit our study to the convergence of the solution sets in the image space. If we aim at a direct extension of stability results developed in the special framework of vector optimization, in order to study the convergence of the whole solution set in the image space of our set optimization problem, that is in general a collection of sets, we need to define the convergence of a sequence of collections of sets. Our approach avoids a direct definition of the convergence of sequences of collections of sets since it implicitly formulates the idea of upper and lower convergence of the solution of a set optimization problem by means of the classical notions of set convergences in the sense of Kuratowski-Painleve and Hausdorff. Indeed, we obtain "lower convergence" or "internal stability" results by requiring that each set in the solution of the original set optimization problem is the limit of a converging sequence of solutions of perturbed problems. Moreover, we consider a formulation of "upper convergence" or "external stability" by requiring that the limit of a converging sequence of solutions of perturbed problems belongs to the solution of the original problem. Under suitable continuity requirements on the set-valued objective map, we obtain both internal and external stability results in the image space.

It is well-known that, in vector optimization, the upper limit of a sequence of solutions of perturbed problems is weakly minimal. Hence, it is not surprising that in order to prove our external stability results we refer to a notion of weak minimality in set optimization. Indeed, we prove that the limit, in the image space, of a sequence of weakly minimal sets of perturbed set optimization problems is a weakly minimal solution of the original problem.

The inner stability of the solutions in the image space, i.e. the possibility to obtain each minimal set of the original problem as the limit of a sequence of minimal sets of perturbed problems, is studied under appropriate compactness assumptions involving the whole collection of solutions of the perturbed problems in the image space. Such a strong assumption can be avoided if we strengthen the lower continuity property of the set-valued objective map by

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requiring cone lower Hausdorff continuity instead of the classical lower semicontinuity. As a consequence, the convergence result works for the conical extension in the image space of the solution of the original problem.

References

[1] Gutierrez C., Miglierina E., Molho E. Novo, V.: Convergence of solutions of a set optimization problem in the image space, Journal of Optimization Theory and Applications, 170, 358-371 (2016).

[2] Kuroiwa, D.: On set-valued optimization. Nonlinear Anal. 47, 1395-1400 (2001).

[3] Lucchetti, R., Miglierina, E.: Stability for convex vector optimization problems. Optimization 53, 517-528 (2004).

[4] Miglierina, E., Molho, E.: Convergence of minimal sets in convex vector optimization. SIAM J. Optim. 15, 513-526 (2005).