Second order dynamical systems with penalty terms associated to monotone inclusions

Szilárd László, R.I. Boţ, E.R. Csetnek *

Abstract

In this paper we investigate in a Hilbert space setting a second order dynamical system of the form

$$\ddot{x}(t) + \gamma(t)\dot{x}(t) + x(t) - J_{\lambda(t)A}\big(x(t) - \lambda(t)D(x(t)) - \lambda(t)\beta(t)B(x(t))\big) = 0,$$

where $A : \mathcal{H} \rightrightarrows \mathcal{H}$ is a maximal monotone operator, $J_{\lambda(t)A} : \mathcal{H} \to \mathcal{H}$ is the resolvent operator of $\lambda(t)A$ and $D, B : \mathcal{H} \to \mathcal{H}$ are cocoercive operators, and $\lambda, \beta : [0, +\infty) \to (0, +\infty)$, and $\gamma : [0, +\infty) \to (0, +\infty)$ are step size, penalization and, respectively, damping functions, all depending on time. We show the existence and uniqueness of strong global solutions in the framework of the Cauchy-Lipschitz-Picard Theorem and prove ergodic asymptotic convergence for the generated trajectories to a zero of the operator $A + D + N_C$, where $C = \text{zer}B$ is the set zeroes of the operator B and N_C denotes the normal cone operator of C. To this end we use Lyapunov analysis combined with the celebrated Opial Lemma in its ergodic continuous version. Furthermore, we show strong convergence for trajectories to the unique zero of $A + D + N_C$, provided that A is a strongly monotone operator.

References

[1] R.I. Boţ, E.R. Csetnek, S.C. László, Second order dynamical systems with penalty terms associated to monotone inclusions, (submitted)

*Technical University of Cluj-Napoca