

# On directional derivative for cone-convex functions

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## Abstract

Abstract: Let  $X$  be a linear space. Let  $Y$  be a Banach space and let  $K \subset Y$  be a closed convex cone. We consider vector-valued functions  $f : X \rightarrow Y$ . We study existence of directional derivatives for  $K$ -convex functions. In [1] it was proved that the existence of directional derivative of cone-convex function  $f : X \rightarrow Y$  implies the weak sequential completeness of the space  $Y$ . In [2] we study isomorphisms of a Banach space  $Y$  with the space  $c_0$ . We get

**Theorem.** Let  $X$  be a linear space and  $Y$  be a Banach space. Let  $K \subset Y$  be a closed normal and convex cone. If for every  $K$ -convex function  $F : X \rightarrow Y$  there exist directional derivatives for every  $h \in X$  at  $x_0 = 0$ , then there is no subspace in  $Y$  isomorphic to  $c_0$ .

As a corollary we get a characterization of weak sequential complete Banach spaces in terms of existence of directional derivative for Banach lattices.

## References

- [1] E. Bednarczuk, K. Leśniewski, *On weakly sequentially complete Banach spaces*, Journal of Convex Analysis, vol. 24 (2017) No 4.
- [2] K. Leśniewski, *On directional derivatives for cone-convex functions*, Mathematical Methods of Operations Research, in review.

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