A Pontryagin-type Maximum Principle including Transversality Conditions for non-linear Infinite Horizon Optimal Control in Hilbert Spaces

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Abstract

In this talk, we address a class of infinite horizon optimal control problems with vector-valued states and controls involving the Lebesgue integral in the objective and a non-linear dynamics. These problems arising in many economic, biological and technical models. Often, optimal control problems with distributed time horizon can be modelled in this problem setting, as well.

The following abstract problem is studied: Minimize the objective function

$$J(x,u) = \int_0^\infty r(t, x(t), u(t))\mathrm{d}t$$

$$\text{w.r.t. } x \in \mathbb{X},\ u \in \mathbb{U}$$

fulfilling the non-linear differential equation

$$\dot{x}(t) = \varphi(t, x(t), u(t)), \quad x(0) = x_0\,.$$

Usually, for $\mathbb{X}$ the space of locally absolutely continuous functions $AC_{\mathrm{loc}}((0,\infty))$ is chosen. The key idea in our approach is to embed AC_{loc} into an appropriate weighted Sobolev space $W_2^1((0,\infty),\mu)$. Thus the optimization becomes a formulation in $\mathbb{X} \times \mathbb{U} = W_2^1((0,\infty),\mu) \times L_2((0,\infty),\mu)$.

As the main result we prove a Pontryagin-type maximum principle. We observe that the arising co-state in this principle belongs to the space $W_2^1((0,\infty),\mu^{-1})$. This implies a natural transversality condition. This condition brings advantages for numerical calculations, especially indirect pseudospectral methods.

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