Optimal investments under uncertainty

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In a classical Markowitz portfolio optimization problem an investor seeks to minimize the variance of portfolio return given a lower bound on its expected return. From a practical point of view, one of the gravest deficiencies of the Markowitz model is the sensitivity of optimal portfolio weights to parameters: the mean and the covariance matrix of asset returns. These are subject to estimation errors and result, in practice, in very unstable portfolio recommendations. The Markowitz model is also unable to incorporate investor's forecasts in the construction of optimal portfolios, a feature demanded by practitioners.

A break-through came with an idea due to Black and Litterman at the turn of the 1990s. Their model combines statistical information on asset returns with investor's views within the Markowitz mean-variance framework. The main assumption underlying the Black-Litterman model is that asset returns and investor's views are multivariate normally distributed. However, the empirical research demonstrates that the distribution of asset returns has fat tails and is asymmetric, which contradicts normality. Moreover, recent advances in risk measurement advocate replacing the variance by risk measures that take account of tail behavior of the portfolio return distribution. We extend Black-Litterman theory into general continuous distributions with the risk measured by deviation measures. In the present study we build on achievements of Rockafellar and coworkers to characterize an equilibrium distribution as a solution of an inverse optimization problem when the risk is measured by deviation measures. Analytical formulas are derived for elliptical distributions and efficient numerical methods are designed for general distributions. We introduce a quantitative model for stating investor's views and blending them consistently with the market information via Bayes formula.

Theoretical advances of the model are complemented by numerical examples in which we explore the effect that the choice of distribution makes on posterior distribution and optimal portfolio weights. We demonstrate that both quantities are significantly affected by the choice of the type of prior distribution and the distribution of investor's views. This exposes the weakness of the multivariate normal assumption made in the Black-Litterman procedure and shows that one should not promote simplicity of original formulas over the correct choice of distribution for modeling of market risk factors.