Cooperative vs. non-cooperative approach to differential R&D game: Endogenous uncertainty as the new market inefficiency source

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In this paper I make use of some recent findings in piecewise-smooth dynamical systems,(Di Bernardo, Budd, Champneys, Kowalczyk, Nordmark, Olivar, and Piiroinen 2008) and (Colombo and Jeffrey 2011) and apply them to the standard framework of R&D differential game like (Bondarev 2016) along with learning-by-doing effects as modelled in (Greiner and Bondarev 2017).

The framework consists of arbitrary number of R&D firms maximizing revenue from innovations. They are engaged both in quality-improving innovations for existing products and in introduction of new products to the market. While firms jointly develop the range of new products, they independently develop all existing products. Thus every firm solves an infinite-dimensional optimal control problem and the solution is characterized as an open-loop Nash non-cooperative one. Due to learning-by-doing effect experienced by every firm in development of newer products, there exist multiple (1 or 3) equilibria for such a game, while due to knowledge spillovers and potential change in the technological leadership from one firm to the other (which may happen multiple times) the resulting dynamical system is of piecewise-smooth type.

The analysis starts with the case ot 2 firms. which can be illustrated with the help of 3-dimensional phase space and then is extended to an arbitrary number of firms. It turns out that while for small number of firms the Teixeira singularity is non-generic, the probability of the system to end up in such a regime increases with the number of market participants. This creates a source of endogenous uncertainty, since once number of firms is sufficiently high, there is a non-negligible probability for the market to end up in the technology lock-in around the

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switching leadership regime. However once the centralised solution is considered, this uncertainty disappears, since it is never optimal for the planner to move the system to this technology lock-in. The centralised solution then is represented as a cooperative game among firms and time-consistency of this solution is studied along the lines of (Petrosjan and Zaccour 2003). Then the question of whether the decentralised solution may be switched to cooperative one by some regulative policy is studied.

The paper contributes first to mathematical fields of multi-modal differential games and piecewise-smooth systems analysis. Second, it contributes to the field of R&D games by demonstrating one new potential source of market uncertainty and its impact on the efficiency of market allocations. Third, it contributes to the R&D policy by explicitly comparing cooperative and non-cooperative solution in this rich environment and studying new ways of market regulation corresponding to the emerging uncertainty.

References

Bondarev, A. (2016). Intensity of R&D competition and the generation of innovations in heterogeneous setting. *Journal of Evolutionary Economics 26*(3), 621–653.

Colombo, A. and M. R. Jeffrey (2011). Nondeterministic chaos, and the two-fold singularity in piecewise smooth flows. *SIAM Journal on Applied Dynamical Systems 10*(2), 423–451.

Di Bernardo, M., C. Budd, A. Champneys, P. Kowalczyk, A. Nordmark, G. Olivar, and P. Piiroinen (2008). Bifurcations in nonsmooth dynamical systems. *SIAM Review 50*(4), 629 – 701.

Greiner, A. and A. Bondarev (2017). Optimal r&d investment with learning-by-doing: Multiple steady-states and thresholds. *Optimal Control, Applications and Methods Online first publication*, doi: 10.1002/oca.2301.

Petrosjan, L. and G. Zaccour (2003). Time-consistent shapley value allocation of pollution cost reduction. *Journal of Economic Dynamics and Control 27*(3), 381–398.