Numerical method solving time-optimal differential games with life line

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The differential game theory is a branch of mathematics, which has been extensively studied theoretically. The existence and uniqueness of the value function were known to be proved for a sufficiently broad class of problems. Analytical methods used to construct the value function have proven efficiently; however, they can hardly be applied even to basic problems. Therefore, development and implementation of numerical methods for constructing the value function are important. This presentation deals with time-optimal differential games. In such games, the first player tries to lead the system to a prescribed target set as soon as possible, and the second one hinders this. In the framework of this class of games, many practical pursuit-evasion problems are formalized and solved.

A numerical grid method based on the viscosity solution concept for the corresponding Hamilton-Jacobi equation and dynamic programming principle was suggested by M. Bardi, M. Falcone, and their colleagues. A theoretical proof justifies this method for constructions in an infinite grid covering the entire game space. However, in practical computations, only a finite grid can be used, thus describing the function in some bounded part of the game space.

During the talk the authors will show that, actually, the problem to solve by the discussed method is in the class of time-optimal games with a life line and prove that this method converges to the generalized solution of the corresponding Hamilton-Jacobi equation. Also it is planned to present results on a proof of a theorem regarding existence sufficient conditions of generalized solution for such games, the existence of the value function and justification of their coincidence. Additionally, this talk will include several numerical examples.

References

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