Linear quadratic game of exploitation of common renewable resources with inherent constraints

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Abstract. In this paper, we analyse a linear quadratic multistage game of extraction of a common renewable resource—fishery—by many players with inherent state dependent constraints for exploitation and infinite time horizon. To the best of our knowledge, such games have never been studied. We analyse social optimum and Nash equilibrium for feedback information structure and compare the results obtained in both. For Nash equilibria, we obtain a value function that is contrary to intuitions from standard linear quadratic games. In our games, social optimum always results in sustainability, while Nash equilibrium leads to depletion of the fishery in finite time for realistic levels of initial biomass of fish. Therefore, we also study introduction of a tax in order to enforce socially optimal behaviour of the players. Besides, this game constitutes a counterexample to simplifications of techniques often used in computation of Nash equilibrium and/or optimal control.

1. Summary of state-of-the-art and the new contributions of the paper

In this paper we present an analysis of a discrete time dynamic game of extraction of common renewable resource—fishery—by many players with infinite time horizon and possibility of depletion. The game is defined such that increasing number of players does not mean introduction of additional users of the resource, but decomposition of the decision making structure of the same mass of users (into regions, countries, firms). The game is linear-quadratic with constraints which are inherent to the problem: player cannot extract negative amount or more than available. Such an obvious modification of the standard framework of linear quadratic game substantially changes both Nash equilibrium and social optimum. We look for Nash equilibria and social optima in feedback form (strategies dependent on the state of the resource).

After modification, we can calculate social optima for arbitrary number of players and prove that they are identical, while Nash equilibrium can be computed only for continuum of players, and we obtain piecewise linear equilibrium with surprisingly compound value function resulting from exhaustion: nonsmooth, piecewise linear-quadratic with infinitely many intervals. For finitely many players a negative result can be proved, that the equilibrium is not even piecewise linear while the value function piecewise linear-quadratic with less than three intervals.

In obtained results, we always have sustainability at social optima and exhaustion in finite time for reasonable interval of initial states for Nash equilibria.

When applicational aspect is considered, we also calculate tax rate enforcing social optimum, and, therefore, sustainability, in the game with continuum of players. This is a Pigovian taxation, of purely regulatory purpose.

Besides, during the process of computation of social optimum, we discovered that this simple dynamic optimization problem constitutes a counterexample to correctness of skipping or relaxation of checking terminal condition—a simplification often used in applications.