

Subject: Extended Abstract prepared for the 11th International ISDG Workshop

Title: The Recursive Bargaining Solution for NTU Differential Games

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I present a novel solution mechanism for  $n$ -person noncooperative NTU differential games. It is shown that the so called recursive bargaining solution increases efficiency against the noncooperative Nash equilibrium. The crucial difference to the standard cooperative approach<sup>1</sup> is that agents do not mutually agree to maximize overall payoffs and distribute them appropriately, but that they bargain over the controls.

The idea is, that all agents meet before the game has started and bargain over stationary Markovian strategy profiles. To define a bargaining solution we first set up a bargaining problem. A *bargaining problem* is a tuple of noncooperative Nash strategies and a set of feasible controls. Controls are feasible if the implementation yields a higher value than the noncooperative Nash value. An *agreeable bargaining solution* is then defined as a mapping from the family of bargaining problems into the set of feasible controls.

Since agreement is not enforced by a binding contract, we need to ensure that the agents stick to the agreed upon bargaining solution. Unilateral deviations from the bargaining agreement is punished by a grim trigger strategy. We consider games with almost perfect state information. The agents thus observe if the state evolves according to the bargaining solution or if at least one agent has deviated. If deviation occurs, all agents stick to the noncooperative Nash equilibrium strategies.

Under the framework specified above we can show that there always exists a recursive bargaining equilibrium. That is, there always exists a bargaining solution which payoff dominates the noncooperative Nash strategies *and* that the bargaining solution is dynamically stable (therefore the recursiveness). In particular I can show that the Nash (1950) bargaining solution implements Pareto efficient controls.

#### References:

Nash, J. F., Jr. (1950): "The Bargaining Problem". *Econometrica* 18 (2).

Yeung, D.; Petrosyan, L. (2016): *Subgame Consistent Cooperation: A Comprehensive Treatise*, Springer.

Zaccour, G. (2008): "Time Consistency in Cooperative Differential Games: A Tutorial", *INFOR: Information Systems and Operational Research* 46 (1) pp. 81 – 92.

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<sup>1</sup>See e.g. Yeung and Petrosjan (2016) and Zaccour (2008).