BRANCHING WITHIN BRANCHING: A GENERAL MODEL FOR HOST PARASITE EVOLUTION

GEROLD ALSMEYER (AND SÖREN GRÖTTRUP)

We consider a general host-parasite model for a cell population and proliferating parasites colonizing these cells. It can be viewed as an extension of a model studied by Bansaye [3]. Cells multiply in accordance with an ordinary Galton-Watson process. Morover, the multiplication mechanisms of cells and parasites obey some dependence structure since cells and parasites influence each other's reproduction in real biological settings. More precisely, the number of daughter cells determines the reproduction law of a parasite living in the mother cell and the way it shares its offspring to these daughter cells. We present equivalent integral conditions for almost sure extinction of parasites by making use of a strong relation of this event to the behavior of parasite multiplication along a randomly picked cell line through the cell tree, the latter forming a branching process in random environment. We also provide results on the asymptotic behavior of the branching within branching process in the case when parasites survive. In particular, we look at the processes of contaminated cells and of parasites by using martingale theory. For both processes, we prove Kesten-Stigum type results and present equivalent integrability conditions for the martingale limit to be positive with positive probability. The case when these conditions are violated is also considered, and we determine exponential growth rates of the two processes in these cases. For the process of contaminated cells, we show that a proper (Heyde-Seneta) normalization exist, such that the limit is nondegenerate.

References

[1] G. Alsmeyer and S. Gröttrup. A host-parasite model for a two-type cell population. *Adv. in Appl. Probab.*, 45(3):719–741, 2013.

[2] G. Alsmeyer and S. Gröttrup. Branching within branching: a model for host-parasite co-evolution. *Stochastic Process. Appl.*, 126(6):1839–1883, 2016.

[3] V. Bansaye. Proliferating parasites in dividing cells: Kimmel's branching model revisited. *Ann. Appl. Probab.*, 18(3):967–996, 2008.

[4] M. Kimmel. Quasistationarity in a branching model of division-within-division. In *Classical and modern branching processes (Minneapolis, MN, 1994)*, volume 84 of *IMA Vol. Math. Appl.*, pages 157–164. Springer, New York, 1997.