PHASE TRANSITION FOR THE VARIANCE OF THE ℓ_p^n-NORM OF THE GAUSSIAN VECTOR

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We study the variance of the ℓ_p^n-norm $||G||_p$ of the standard Gaussian vector G in $\mathbb{R}^n$ in the regime when p grows to infinity with n. It is known that for a fixed $p < \infty$, $\mathbf{Var}\|G\|_p \simeq v_p n^{2/p-1}$, where v_p depends only on p and not on n, while the variance of the $\|\cdot\|_\infty$–norm of G is of order $(\log n)^{-1}$. In [1], Paouris, Valettas and Zinn considered, in particular, the case when p grows to infinity with n and showed that $\mathbf{Var}\|G\|_p \simeq \frac{2^p}{p} n^{2/p-1}$ for $p \leq c \log n$ and $\mathbf{Var}\|G\|_p \simeq \frac{1}{\log n}$ for $p \geq C \log n$ ($C, c > 0$ being universal constants). This result leaves the gap $c \log n \leq p \leq C \log n$ in which the behaviour of the variance was not clarified. We resolve this issue and determine the "phase transition window" in which the variance $\mathbf{Var}\|G\|_p$ changes from polynomially small in n to logarithmic. We also discuss an application of our result to Dvoretzky's theorem for ℓ_p^n.

This is a joint work with Konstantin Tikhomirov.

References

[1] Grigoris Paouris, Petros Valettas, and Joel Zinn, *Random version of Dvoretzky's theorem in ℓ_p^n*. Stochastic Processes and their Applications (2017). arXiv:1510.07284