STOCHASTIC RECURSIONS: BETWEEN KESTEN'S AND GREY'S ASSUMPTIONS

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We study the stochastic recursions $R_n = A_n R_{n-1} + B_n$ and $R_n = \max\{A_n R_{n-1}, B_n\}$, where $(A_n, B_n) \in \mathbb{R}\times\mathbb{R}$ is an i.i.d sequence of random vectors and R_0 is an arbitrary initial distribution independent of $(A_n, B_n)_{n\geq 1}$. The tail behavior of their stationary solutions R is well known under the so called Kesten-Grincevićius-Goldie or Grey conditions. Under these assumptions, the tail of R is determined by A or B alone. It is natural to go a step further and to ask what happens when we are in a heavy tail regime, but neither Kesten's nor Grey's assumptions are satisfied. Is this "in-between" case much different from the known ones? The answer is affirmative.

In the most simplified version, our basic result says that if $A \geq 0$ a.s., $\mathbb{E}\log A < 0$ and there exists $\alpha > 0$ such that $\mathbb{E}A^\alpha = 1$, $\rho := \mathbb{E}A^\alpha \log A < \infty$, and $L(x) := x^\alpha \mathbb{P}(B > x)$ is a slowly varying function (supplemented with some more technical assumptions), then

$$x^\alpha \mathbb{P}(R > x) \sim \frac{1}{\rho}\int_0^x \frac{L(t)}{t}\mathrm{d}t = \frac{\mathbb{E}B^\alpha \mathbf{1}_{0<B\leq x}}{\alpha\rho} \qquad \text{as } x \to \infty.$$

Under suitable conditions on A and B we find also the second order asymptotics of the tail of R. To obtain these results we prove renewal theorems that essentially generalize existing ones and are of independent interest.

This talk is based on joint work with Ewa Damek [1].

References

[1] E. Damek, B. Kołodziejek Stochastic recursions: between Kesten's and Grey's assumptions *arXiv:1701.02625*, 1–25, 2017.