

# INSTANTANEOUS BLOWUP, OLD AND NEWS

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Consider the heat equation

$$\frac{\partial u}{\partial t} = \Delta u + V(x)u$$

for  $x \in \mathbb{R}^N$  with a positive potential  $V(x)$ . If  $V$  is "too singular", then this equation may not have any positive solutions, as was discovered in 1984. We shall prove the sharp version of this result and discuss later developments, including new results obtained in 2016-17. The Euclidean space  $\mathbb{R}^N$  can be replaced by the Heisenberg group  $\mathbb{H}^N$  and other Carnot groups, and the heat equation can be replaced by the Ornstein-Uhlenbeck equation and others as well. Some nonlinear results will be mentioned. A key role is played by scaling. Many coauthors are involved.