SHARP ESTIMATES FOR DIRICHLET HEAT KERNELS IN $\mathbb{R}^n$

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Let $g_n(t,x,y) = (4\pi)^{-n/2}e^{-|x-y|^2/4t}$ be the standard Gaussian heat kernel, and let $g_n^D(t,x,y)$ be the Dirichlet heat kernel for a given set $D \subset \mathbb{R}^n$. In 2002 Qi S. Zhang [4] has proven that for every bounded $C^{1,1}$ set D there are constants $C, c_1, c_2 > 0$ such that

$$\frac{1}{C}\left(1 \wedge \frac{\delta(x)\delta(y)}{t}\right)(4\pi)^{-n/2}e^{-c_1|x-y|^2/4t} \le g_n^D(t,x,y)$$
$$\le C\left(1 \wedge \frac{\delta(x)\delta(y)}{t}\right)(4\pi)^{-n/2}e^{-c_2|x-y|^2/4t},$$

where $x, y \in D$, $t < T$. In fact, the upper bound is due to E.B Davies [1]. A natural question is if we could take $c_1 = c_2 = 1$. The estimates would be then much more precise and easy to describe by means of the global heat kernel $g_n(t,x,y)$. It appears that even for convex sets this may be not possible. However, studies on some particular cases have shown that sometimes it is enough to change nonexponential factors, e.g. the following bounds were obtained in [3] for the interval $(0,1) \subset \mathbb{R}$:

$$\frac{1}{C}\left(1 \wedge \frac{xy}{t}\right)\left(1 \wedge \frac{(1-x)(1-y)}{t}\right) g_1(t,x,y) \le g_1^{(0,1)}(t,x,y)$$
$$\le C\left(1 \wedge \frac{xy}{t}\right)\left(1 \wedge \frac{(1-x)(1-y)}{t}\right) g_1(t,x,y),$$

were $x, y \in (0,1)$, $t < T$.

During the talk we will discuss the problem for other sets. In particular, we will focus on the case of n-dimensional ball, which was studied in the recent paper [2].

References

[1] E. B. Davies, *The equivalence of certain heat kernel and Green function bounds.* J. Funct. Anal., 71 (1987), 88–103.

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[3] A. Pyć, G. Serafin, T. Żak, *Supremum distribution of Bessel process of drifting Brownian motion.* Probab. Math. Statist. 35.2 (2015), 201-222.

[4] Q. S. Zhang, *The boundary behavior of heat kernels of Dirichlet Laplacians.* J. Differential Equations, 182(2002), 416–430.