FREE INFINITE DIVISIBILITY FOR R-DIAGONAL DISTRIBUTIONS

KAMIL SZPOJANKOWSKI

We say that the distribution of a non-commutative and in general non-normal random variable a is R-diagonal if

$$\kappa_n(a_1, \ldots, a_n) = 0$$

whenever $a_1, \ldots, a_n \in \{a, a^*\}$ are not alternating in a and a^*. Where by $(\kappa_n)_{n\geq 1}$ we denote the free cumulant functionals.

The class of R-diagonal $*$-distributions is fairly well understood in free probability. It was introduced in [3] and sice then it has received quite a bit of attention in the free probability literature. In particular, elements with R-diagonal $*$-distributions were among the first examples of non-normal elements in W^*-probability spaces for which the so-caled 'Brown spectral measure' was calculated explicitly (in [2]), and for which the Brown measure techniques could be used to find invariant subspaces (in [4]).

In this class, we consider the concept of infinite divisibility with respect to the operation $\boxplus$ of free additive convolution. One way to approach $\boxplus$-infinite divisibility is to construct a (suitable for $*$-distributions) version of a bijection constructed in [1] which relates free independence to another form of noncommutative independence, namely Boolean independence. In the next step we introduce the concept of an *η-diagonal distribution* that is the Boolean counterpart of an R-diagonal distribution. We establish a number of properties of η-diagonal distributions, then we examine the canonical bijection relating η-diagonal distributions to infinitely divisible R-diagonal ones. The overall result is a parametrization of an arbitrary $\boxplus$-infinitely divisible R-diagonal distribution that can arise in a C^*-probability space, by a pair of compactly supported Borel probability measures on $[0, \infty)$.

As an application of the parametrization we prove that the set of $\boxplus$-infinitely divisible R-diagonal $*$-distributions is closed under the operation $\boxtimes$ of multiplicative convolution.

The talk is based on joint work with H. Bercovici (Indiana University, USA), A. Nica (University of Waterloo, Canada) and M. Noyes (Bard College, USA).

References

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