STABILITY OF HEAT KERNEL ESTIMATES FOR DIFFUSIONS WITH JUMPS UNDER NON-LOCAL FEYNMAN-KAC PERTURBATIONS

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Suppose X is a Hunt process on $\mathbb{R}^d$ that has a jointly continuous transition density function $p(t,x,y)$ with respect to the Lebesgue measure and that the following two-sided heat kernel estimates hold for $p(t,x,y)$ on $(0,1]\times\mathbb{R}^d\times\mathbb{R}^d$.

$$C^{-1}(\Gamma_{c_2}(t;x-y)+\eta(t;x-y)\le p(t,x,y)\le C(\Gamma_{c_4}(t;x-y)+\eta(t;x-y), \qquad (1)$$

where

$$\Gamma_\lambda(t;x):=t^{-d/2}e^{-\lambda|x|^2/t} \quad \text{and} \quad \eta(t;x):=\frac{t}{(t^{1/2}+|x|)^{d+\alpha}}.$$

We show that, with certain Kato class conditions on μ, F, the following non-local Feynman-Kac transform

$$T_t^{\mu,F}f(x)=\mathbb{E}_x\left[\exp\left(A_t^\mu+\sum_{s\le t}F(X_{s-},X_s)\right)f(X_t)\right]$$

has a heat kernel $q(t,x,y)$ and $q(t,x,y)$ has two-sided estimates (1) on $(0,1]\times\mathbb{R}^d\times\mathbb{R}^d$ but with a different set of possible different constants. Chen-Kim-Song discussed the stability of Dirichlet heat kernel estimates for jumping processes under Feynman-Kac transform in [1]. The process discussed here is a mixture of diffusions and jumping processes, and the Gaussian bounds in (1) have different constants c_2, c_4 in the exponents for the upper and lower bound estimates.

References

[1] Z-Q. Chen, P. Kim and R. Song, Stability of Dirichlet heat kernel estimates for non-local operators under Feynman-Kac perturbation, *Trans. Amer. Math. Soc.* **367** (2015), 5237-5270.

[2] Z-Q. Chen, L. Wang, Stability of Heat Kernel Estimates for Diffusions with Jumps under Non-local Feynman-Kac Perturbations. arXiv:1702.04489 (2017)