PASSAGE TIMES OF ADDITIVE RANDOM WALKS AND ENUMERATION FORMULAS OF MULTITYPE PLANE FORESTS.

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It has recently been proved in [1], that the genealogy of a d-types Bienaymé-Galton-Watson forest can be encoded by an additive random walk of the type,

$$\mathbf{S}_{\mathbf{n}} = S^{(1)}_{n_1} + \cdots + S^{(d)}_{n_d}, \quad \mathbf{n} = (n_1, \ldots, n_d) \in \mathbb{N}^d,$$

where $S^{(i)} = (S^{i,j}, j \in [d])$, $i \in [d]$ are d independent $\mathbb{Z}^d$-valued random walks such that $S^{i,j}$ are increasing for $i \neq j$ and downward skip free for $i = j$. This process extends the Lukasiewicz-Harris path of single type forests and satisfies a multivariate ballot theorem. It allows us to determine the law of some functionals such as the total progeny of the forest or the number of vertices with a given degree. We also obtain some enumeration formulas of multitype plane forests. In particular, we determine the number of unlabeled forests when the number of vertices of type j who have a parent of type i is given. We also enumerate forests for which the number of vertices of each indegree type is given. Then we recover some recent enumeration results of multitype labeled forests as consequences of the multivariate ballot theorem. This theorem is also applied to obtain a new proof of the (multivariate) Lagrange-Good inversion formula.

References

[1] Loïc Chaumont and Rongli Liu: Coding multitype forests: Application to the law of the total population of branching forests. *Trans. Amer. Math. Soc.* 368, no. 4, 2723–2747, (2016).