COMPARISON OF WEAK AND STRONG MOMENTS FOR VECTORS WITH INDEPENDENT COORDINATES

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We will try to tackle the following problem: "Characterize random vectors for which weak and strong moments are comparable." As it turns out, such a comparison holds for vectors with independent coordinates with α-regular growth of moments:

Theorem 1. *Let $X_1, \ldots, X_n$ be independent mean zero random variables with finite moments such that*

$$\|X_i\|_{2p} \leq \alpha \|X_i\|_p \qquad \text{for every } p \geq 2 \text{ and } i = 1, \ldots, n, \tag{1}$$

where α is a finite positive constant. Then for every $p \geq 1$ and every non-empty set $T \subset \mathbb{R}^n$ we have

$$\left(\mathbb{E}\sup_{t\in T}\Big|\sum_{i=1}^n t_i X_i\Big|^p\right)^{1/p} \leq C(\alpha)\left[\mathbb{E}\sup_{t\in T}\Big|\sum_{i=1}^n t_i X_i\Big| + \sup_{t\in T}\left(\mathbb{E}\Big|\sum_{i=1}^n t_i X_i\Big|^p\right)^{1/p}\right], \tag{2}$$

where $C(\alpha)$ is a constant which depends only on α.

Moreover, in the case of i.i.d. coordinates (2) implies (1) (with a constant α depending on C only), so the problem posed in the beginning is solved for vectors with i.i.d. coordinates.

We will also discuss the consequences of Theorem 1, such as a deviation inequality for $\sup_{t\in T}|\sum_{i=1}^n t_i X_i|$ and a Khinchine-Kahane type inequality.

The talk will be based on joint work with Rafał Latała [1].

References

[1] R. Latała, M. Strzelecka, *Comparison of weak and strong moments for vectors with independent coordinates*, arXiv:1612.02407