

A NOTE ON THE BIRKHOFF ERGODIC THEOREM

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The classical Birkhoff ergodic theorem states that for an ergodic Markov process the limiting behavior of the time average of a (integrable with respect to the invariant measure) function along the trajectories of the process, starting from the invariant measure, is a.s. constant and equals to the space average of the function with respect to the invariant measure. The crucial assumption here is that the process starts from the invariant measure, which is not always the case. In this talk, under the assumptions that the underlying process is a Markov process on metric space, that it admits an invariant probability measure and that its marginal distributions converge to the invariant measure in the L^1 -Wasserstein metric, we will show that the assertion of the Birkhoff ergodic theorem holds in probability and L^p , $p \geq 1$, for any bounded Lipschitz function and any initial distribution of the process.