RANDOM DIFFERENCE EQUATION AND REGULARLY VARYING TAILS

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Given a sequence of iid two-dimensional random vectors $\{(A_n, B_n)\}_{n\geq 0}$, we will consider a Markov chain defined viz.

$$X_{n+1} = A_{n+1}X_n + B_{n+1}, \quad n \geq 0.$$

Our interests will revolve around asymptotic properties of the stationary distribution of $\{X_n\}_{n\geq 0}$, which satisfies

$$X \stackrel{d}{=} AX + B, \quad X \text{ independent of } (A, B),$$

where (A, B) denotes a generic element of the sequence $\{(A_n, B_n)\}_{n\geq 0}$. We will show, that if $\mathbb{E}[A^\alpha] < 1$,

$$\mathbb{P}[A > t] \sim \log(t)^{-c} t^{-\alpha}, \quad \text{as } t \to \infty$$

with $\alpha > 0$ and $c > 1$ (or more generally $\log^+(A) \in \mathcal{S}(\alpha)$) then under some regularity condition imposed on the vector (A, B),

$$\mathbb{P}[X > t] \sim c_X \log(t)^{-c} t^{-\alpha}, \quad \text{as } t \to \infty.$$

The talk is based on a joint work with Ewa Damek.