ON THE DOMAIN OF GENERATORS OF FELLER PROCESSES

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We study the domain of the generator of stable processes, stable-like processes and more general pseudo- and integro-differential operators which naturally arise both in analysis and as infinitesimal generators of Lévy- and Lévy-type (Feller) operators. In particular we obtain conditions on the symbol of the operator ensuring that certain (variable order) Hölder and Hölder-Zygmund spaces are in the domain. We use tools from probability theory to investigate the small-time asymptotics of the generalized moments of a Lévy or Lévy-type process $(X_t)_{t\geq 0}$,

$$\lim_{t\to 0} \frac{1}{t} \left(\mathbb{E}^x f(X_t) - f(x)\right), \quad x \in \mathbb{R}^d,$$

for functions f which are not necessarily bounded or differentiable. The pointwise limit exists for fixed $x \in \mathbb{R}^d$ if f satisfies a Hölder condition at x. Our results apply, in particular, to stable-like processes, relativistic stable-like processes, solutions of Lévy-driven SDEs and Lévy processes.