OPERATORS RELATED TO THE LAPLACIAN WITH DRIFT IN EUCLIDEAN SPACE

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Let $v \neq 0$ be a vector in $\mathbb{R}^n$. Consider the Laplacian on $\mathbb{R}^n$ with drift $2v$, defined as $\Delta_v = \Delta + 2v \cdot \nabla$. Then Δ_v is self-adjoint with respect to the measure $d\mu(x) = e^{2\langle v,x\rangle}dx$. This measure has exponential volume growth at infinity. We study weak type $(1,1)$ and other sharp endpoint estimates for the Riesz transforms of any order associated with Δ_v, and also for the vertical and horizontal Littlewood-Paley-Stein functions defined in terms of the heat and the Poisson semigroups generated by Δ_v.
The Riesz transforms considered are of type $R_D = D(-\Delta_v)^{-k/2}$, where D is a homogeneous constant-coefficient operator, thus of the form

$$D = \sum_{|\alpha|=k} a_\alpha \partial^\alpha.$$

The operator R_D is of weak type (1,1) with respect to μ if its order k is at most 2. But this condition is not necessary. To give a characterization, we write D as a sum of combinations of differentiations along the vector v and differentiations in directions orthogonal to v. We prove that R_D is of weak type (1,1) with respect to μ if and only if the maximal order of differentiation along v is at most 2.

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