DECOULING INEQALITIES IN BANACH SPACES

STEFAN GEISS (JOINT WORK WITH SONJA COX (AMSTERDAM)

We consider decoupling inequalities for $(\mathcal{F}_n)_{n=0}^{\infty}$ adapted sequences $(d_n)_{n=1}^{\infty}$ of random variables with values in a Banach space X. Our approach is based on restricting the sequences $(d_n)_{n=1}^{\infty}$ according to the conditional distributions of d_n given $\mathcal{F}_{n-1}$. We show that enlargements of filtrations can be done by using arguments from Maurey [4], provide simple representations similar to expansions in terms of Haar functions, and finally prove extrapolation results. A case of particular interest is the *upper decoupling*, where the moments of $\| \sum_{n=1}^{N} d_n \|_X$ are upper bounded by moments of the decoupled sum, say $\| \sum_{n=1}^{N} e_n \|_X$. Here we extend results from [1]. In this case the underlying Banach space X has necessarily to be of finite co-type if one considers any reasonable decoupling where only the laws of the conditional distributions of d_n given $\mathcal{F}_{n-1}$ are restricted (this is shown by adapting an example of Garling [2] and Veraar [5]). However, if one restricts the adapted processes in the way that we can control the 'branching of the conditional distributions' from time-step to time-step in a certain manner, then the upper decoupling works in all Banach spaces, which relates to a result of Klass [3].

References

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