BELLMAN FUNCTION AND MONGE–AMPÈRE EQUATION

VASILY VASYUNIN

The aim of the talk is to present a method of finding solution of a rather general class of extremal problems. This class includes such classical problems as the John–Nirenberg inequality for BMO-functions, the reverse Hölder inequality for Muckenhoupt weights, and many others. The mentioned class describes the case when the Bellman equation is reduced to the homogeneous Monge–Ampère equation for two variables and the domain of the definition of the Bellman function is the difference of two concave planar domains. The process of finding the Bellman function of an extremal problem (i.e. solving the boundary value problem of the Monge–Ampère equation) is reduced to constructing the so-called foliation of the domain. We classify all possible local foliations and describe the way how to construct the global foliation from these local elements. The main parameter determining the global architecture of the foliation is the sign of the curvature of the boundary curve. We describe the evolution of the foliation when the domain is increasing. A detailed paper (153 pages) devoted to a special case of this construction for the case of BMO space is accepted for publishing in "Memoirs of the American Mathematical Society" (for a preprint version see arXiv:1510.01010). A paper describing the general case is now in preparation.

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