NUMERICAL RANGE OF NON-HERMITIAN RANDOM GINIBRE MATRICES AND THE DVORETZKY THEOREM

KAROL ŻYCZKOWSKI

For any matrix A of order N one defines its *numerical range* (field of values) as a subset of the complex plane, $W(A) = \{z \in \mathbb{C} : \langle\psi|A|\psi\rangle = z,\ |\psi\rangle \in \mathcal{H}_N,\ \langle\psi|\psi\rangle = 1\}$. Due to the Toeplitz–Hausdorff this set is convex and if A is normal, it coincides with the convex hull of the spectrum. A short review of properties of numerical range of small matrices is presented and asymptotic results for random matrices are discussed.

Theorem 1. *Consider a random Ginibre matrix G (with independent complex Gaussian entries), of a large size N normalized by the condition $\mathrm{Tr}GG^\dagger = N$, so that its spectrum is asymptotically confined in the unit disk. Then the numerical range $W(G)$ almost surely converges to the disk of radius $\sqrt{2}$.*

This result is shown to be related to the Dvoretzky theorem and the structure of the set of mixed quantum states of size N.

References

[1] B. Collins, P. Gawron, A.E. Litvak and K. Życzkowski, Numerical range for random matrices, J. Math. Anal. Appl. 418, 516–533 (2014).