On the embedding of hedgehogs in function spaces

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Let $S_\omega = \{\infty\} \cup \{(n,m) : n,m \in \omega\}$ be the sequential hedgehog (sequential fan) of spininess ω, where each (n,m) is isolated in S_ω and a basic open neighborhood of ∞ is of the form $N(\varphi) = \{\infty\} \cup \{(n,m) : n \in \omega, m \geq \varphi(n)\}$ for a function $\varphi \in \omega^\omega$.

A space X is said to be Menger [3] if for every sequence $\{\mathcal{U}_n : n \in \omega\}$ of open covers of X, there are finite subfamilies $\mathcal{V}_n \subset \mathcal{U}_n$ such that $\bigcup\{\mathcal{V}_n : n \in \omega\}$ is a cover X. A space X is said to have countable fan-tightness [1] if whenever $A_n \subset X$ and $x \in \overline{A_n}$ $(n \in \omega)$, there are finite sets $F_n \subset A_n$ such that $x \in \overline{\bigcup\{F_n : n \in \omega\}}$. Obviously S_ω does not have countable fan-tightness.

A.V. Archangel'skii [1] proved that every finite power of X is Menger if and only if $C_p(X)$ has countable fan-tightness. Hence, if every finite power of X is Menger, S_ω cannot be embedded into $C_p(X)$.

M. Sakai [4] proved (under CH) that there is Menger space X such that S_ω be embedded into $C_p(X)$. It is answer to Arhangel'skii's problem ([2], Problem II.2.7) under CH.

Let $\mathcal{F} \subseteq [\mathbb{N}]^\infty$ be a semifilter. A space X is $\mathcal{F}$-Menger if for every sequence $\{\mathcal{U}_n : n \in \omega\}$ of open covers of X, there are finite subfamilies $\mathcal{V}_n \subset \mathcal{U}_n$ such that $\{n : x \in \bigcup \mathcal{V}_n\} \in \mathcal{F}$ (see [5]).

For a semifilter $\mathcal{F}$, $S_\mathcal{F} = \{\infty\} \cup \{(n,m) : n,m \in \omega\}$ be the $\mathcal{F}$-hedgehog ($\mathcal{F}$-fan) of spininess ω, where each (n,m) is isolated in $S_\mathcal{F}$ and a basic open neighborhood of ∞ is of the form $N(\varphi) = \{\infty\} \cup \{(n,m) : n \in \omega, m \in \varphi(n)\}$ for function $\varphi : \omega \mapsto \mathcal{F}$.

We will discuss some connections among the $\mathcal{F}$-Menger property and an embedding of $S_\mathcal{F}$ into functional spaces.

References

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